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Control of Chaos in a Nonlinear Prey-Predator Model

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Abstract

The suppresion of chaos in a two-dimensional map, which models the competition between the populations of two species, is presented. As a result, steady-state and different periodic motions, embedded in the chaotic attractor, have been stabilized.

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Inroduction

In the last twenty years the presence of chaos in different domains (physical, chemical, biological or economical) has been extensively demonstrated (see [1], [2] or [3] for a recent survey). The basic characteristic of the chaotic systems, which are dissipative non-linear dynamic systems, defined either as a set of coupled differential equations or as iterative maps, is the extreme sensitivity to small perturbations in their initial conditions. This characteristic has been considered to be a troublesome property and for many years it was generally believed that chaotic behaviors are not controllable. Recent works have demonstrated that even this property permits the use of small perturbations to control phase trajectories of these systems.

An efficient method for achieving control was proposed by Ott, Grebogi and Yorke OGY [4]. They have shown that it is possible to obtain a regular or a periodic behavior by making small time-dependent perturbations in some accesible system parameters. The OGY method and its variants have been applied to different experimental systems [5-7].

Another method of controlling chaos has been recently proposed by Güémez and Matias (G.M.) [8]. This method allows chaotic systems to be stabilized by performing changes in the system variables. It was applied in the case of a chemical system [9], but the authors consider it possibile to apply their method to some biological systems.

In the present work we apply the G.M. method to a model which describes the competition between the populations of two species, a prey-predator model.

The Prey-Predator Model

There are different prey-predator models in the form of a set of coupled differential equations (see e.g. [10]), but we will take into consideration a discrete model defined by the two-dimensional map:

$$\begin{aligned} x_{n+1} &= ax_n (1 - x_n) - bx_n y_n \\ y_{n+1} &= dx_n y_n \end{aligned} \qquad (1)$$

where x and y represent the prey and predator population, respectively, and a, b, d are positive parameters.

This model considers the prey's growth be governed by a logistic map. The terms (−b x y) and (+d x y) describe prey-predator encounters which are favorable to predators and fatal to prey.

The map under consideration has a rich behavior. Thus it has two equilibrium points (fixed points)

$$(x^* = 0, y^* = 0) \text{ and } \left(x^* = \frac{1}{d}, y^* = \frac{a}{b}\left(1 - \frac{1}{d}\right) - \frac{1}{b}\right) \qquad (2)$$

The nonzero fixed point, which depends on parameters a, b, d, is stable for

$$d \in \left(\frac{a}{a-1}, \frac{2a}{a-1}\right) \text{ with } a > 1.$$

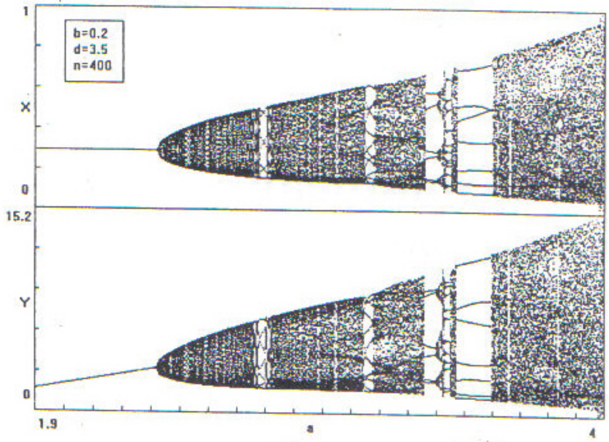

Fig. 1. Bifurcation diagrams for the system's variables.

For $\left(\frac{a}{d} + 2\right)^2 < 4a$ and $a = \frac{d}{d-2}$

the fixed point becomes unstable and begins to bifurcate, the bifurcated solution being oscillatory. This phenomenon is know as a Hopf bifurcation.

For larger values of parameters the limit cycles, obtained by Hopf bifurcations, become unstable and the chaotic behavior of the systems is observed.

The results of numerical investigations are presented below. In Fig. 1 the behavior of the system's variables versus parameter a (with $b = 0.2$ and $d = 3.5$) is shown. A blow-up of a region of $a \in (2.92, 2.94)$ is depicted in Fig. 2, and the details of some windows with different periods are shown in Fig. 3 for $a \in (3.3, 3.6)$.

For $a = d = 3$ the fixed point becomes unstable and begins to bifurcate (a Hopf bifurcation). This can be observed in Figs. 4 and 5 where we plotted the variations of y versus x for two values of a before and after the bifurcation.

In Fig. 6 we show a strange attractor for a value of a in the chaotic domain on Fig. 1.

When y becomes zero, the behavior of the system is a logistic like one. This can be seen on the Fig. 7.

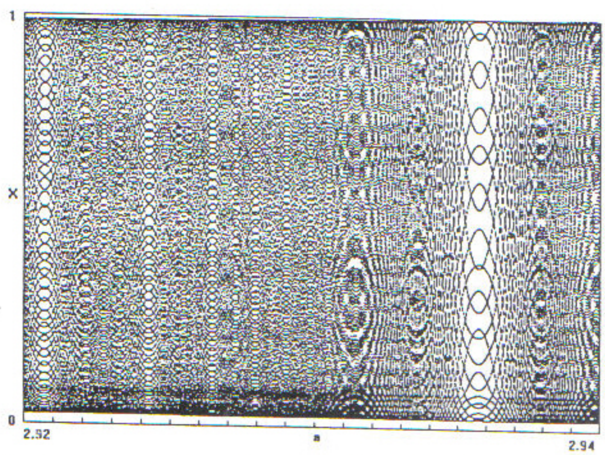

Fig. 2. A blowup of a region of the bifurcation diagram for the x variable.

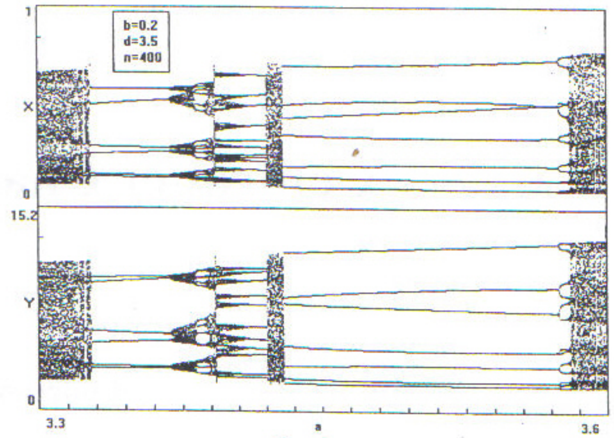

Fig. 3. A blowup of a domain with periodic windows of the bifurcation diagrams.

The Stabilization Method

The G.M. control algorithm, in the case of a two-dimensional map, consists in the application of the pulses in the system variables. Hence the variables x and y are modified, every Δn iterations in the form:

$$x_n \to x_n (1 + \gamma_1) \; ; \; y_n \to y_n (1 + \gamma_2), \qquad (3)$$

where γ_1 and γ_2 represent the intensity of the pulses for x and y, respectively.

For the sake of simplicity we have assumed $\gamma_1 = \gamma_2 = \gamma$.

By this method, depending on the sign of γ, some part of x (or y) is injected or retired from the map, which depends on the value of x_n (or y_n) at that moment. By appropriately choosing Δn and γ, it is possible to stabilize different unstable periodic orbits.

We have applied the G.M. method to the map (1) for some values of the parameters, when the system has a chaotic behavior (a strange attractor). Different values for Δn

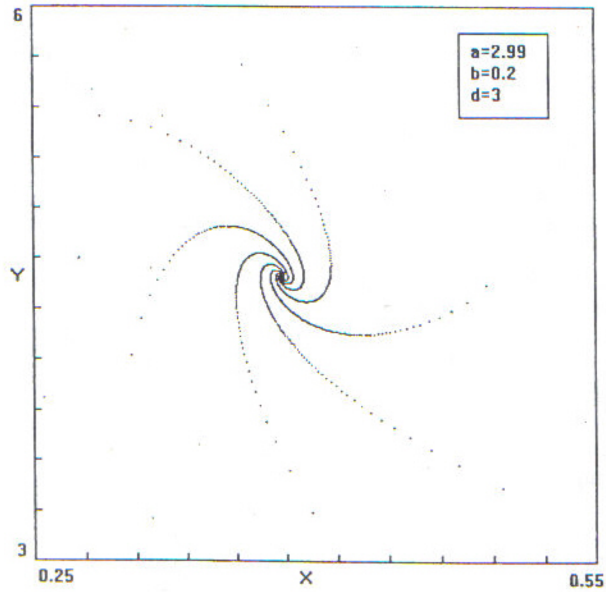

Fig. 4. Phase portrait of the map before the Hopf bifurcation.

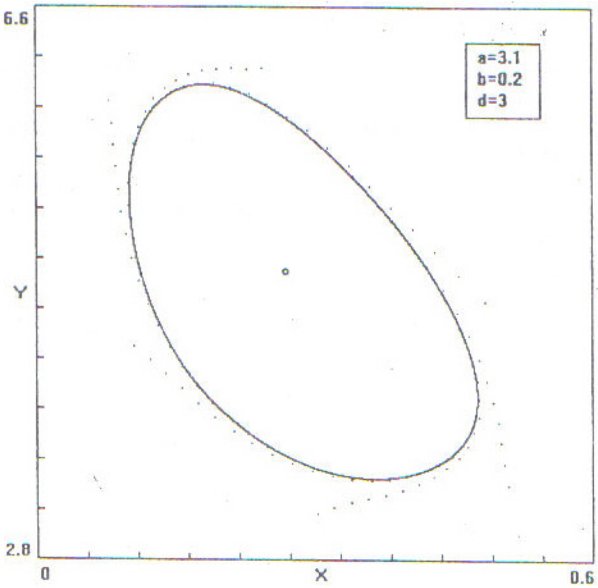

Fig. 5. Phase portrait of the map after the Hopf bifurcation.

and γ have been considered. The results are illustrated in Figs. 8-10, where we have plotted x and y respectively versus number n of iterations. The vertical dashed line splits the figures in two regions: before and after the action of the control algorithm.

Conclusions

This work presents some results of controlling chaos in a two-dimensional map, which models the competition between the populations of two species. We have chosen the G.M. method, which consists in the application of pulses of intesity γ every Δn iterations in the system variables. Thus we were able to stabilize different periodic motions embedded in the chaotic attractor, even a steady state. This result

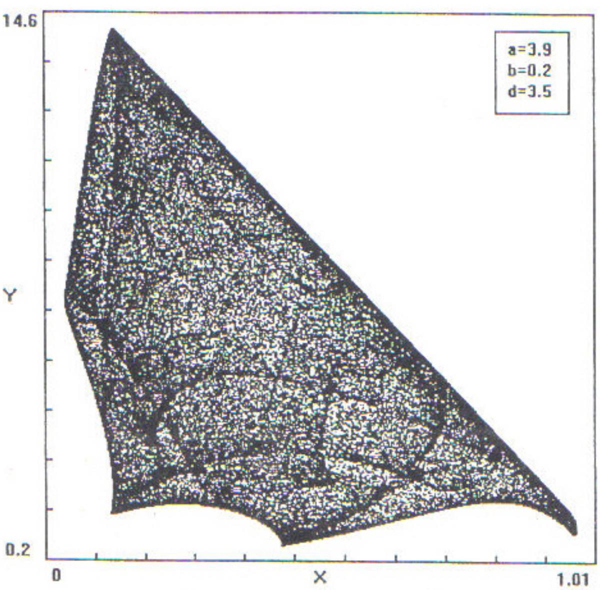

Fig. 6. A strange attractor of the map.

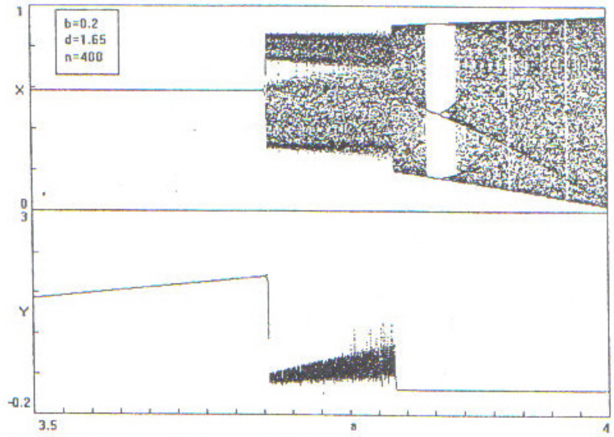

Fig. 7. Behavior of the map when y becomes zero.

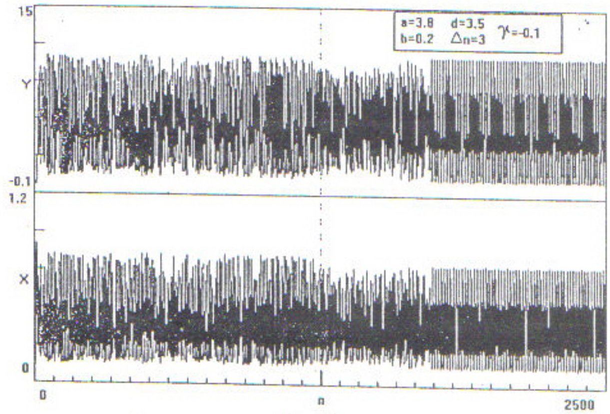

Fig. 8. A stabilized periodic motion of both variables x and y.

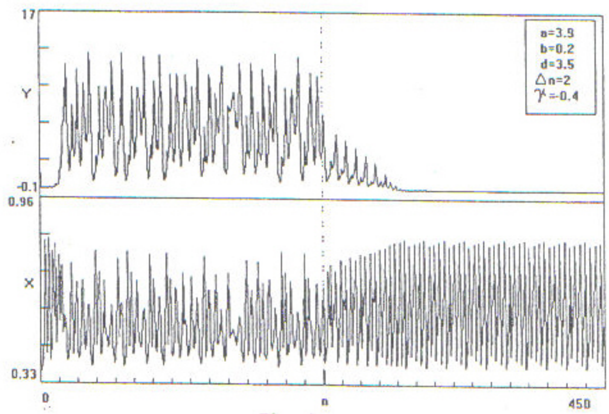

Fig. 9. A stabilized periodic motion of x and a steady-state of y.

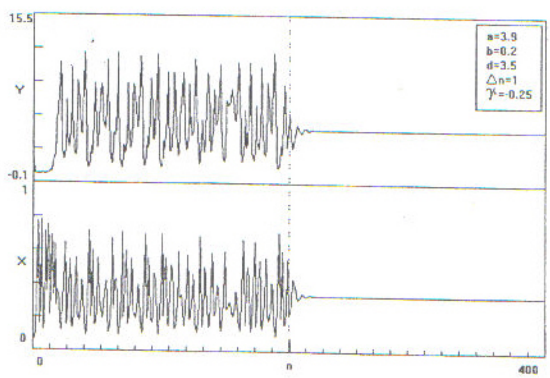

Fig. 10. A stabilized steady-state of x and y.

could be regarded, from a practical point of view, as those which are obtained if one considers (for example) some harvesting activity in the considered biological system.

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